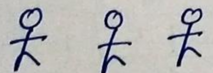
Permutations and Combinations

No. of Arrangements



A B C

3 friends $\Rightarrow$ 3 chairs

$\boxed{}\boxed{}\boxed{}$

$\left.\begin{array}{l} ABC \\ ACB \\ BAC \\ BCA \\ CAB \\ CBA \end{array}\right\} 6$

Discussed in Exercise 7.1, 7.2, 7.3

No. of Selections

A B C

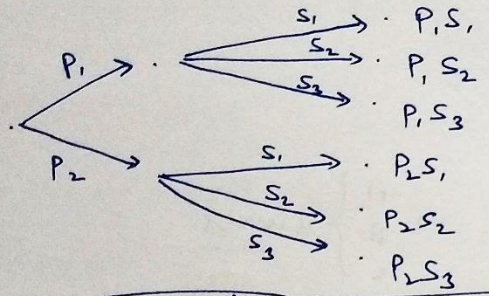
select 1 $\rightarrow$ $\left.\begin{array}{l} A \\ B \\ C \end{array}\right\}$ 3 ways

select 2 $\rightarrow$ $\left.\begin{array}{l} AB \\ BC \\ CA \end{array}\right\}$ 3 ways

select 3 $\rightarrow$ ABC $\}$ 1 way

Discussed in Exercise 7.4

e.g. ① 2 pants → P_1, P_2
3 shirts → S_1, S_2, S_3



$\boxed{\text{Total} = 6 \text{ ways to Dress up}}$

Solution by
Fundamental Principle of Counting
(multiplication rule)

$= \text{Pants} \times \text{shirts}$

$= 2 \times 3$

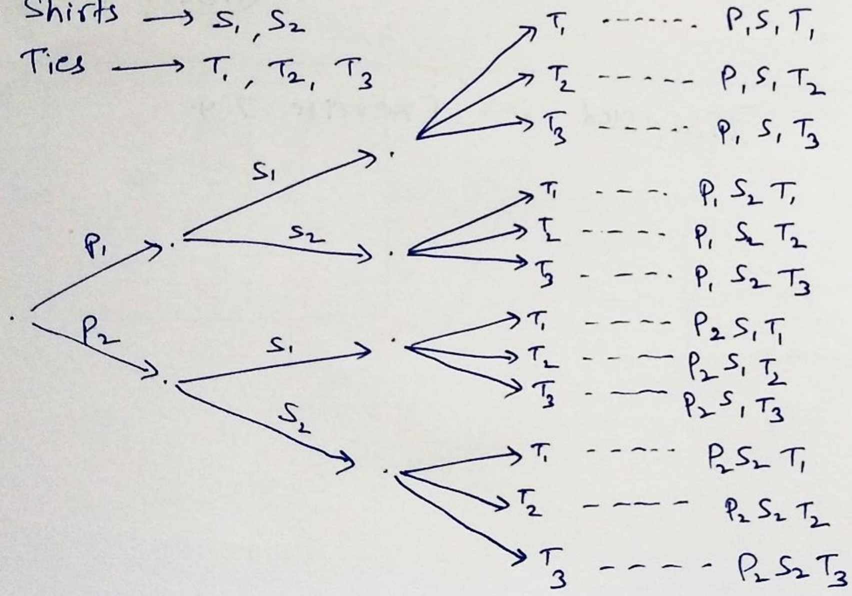
$= 6$

P	S
2	3

×

e.g. ② 2 Pants → P_1, P_2
2 Shirts → S_1, S_2
3 Ties → T_1, T_2, T_3

$\boxed{\text{Total} = 12 \text{ ways}}$



$\text{Pants} \times \text{Shirts} \times \text{Ties}$

No. of ways

P	S	T
2	2	3

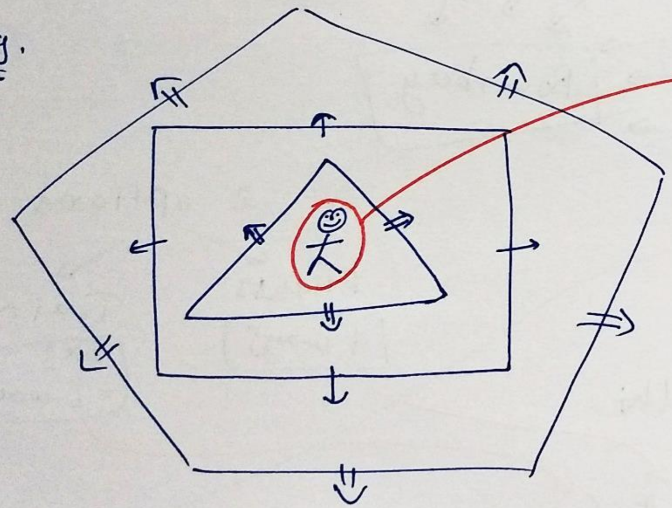
$= 2 \times 2 \times 3$

$= 12$

Fundamental Principle of Counting :
Theorem

(I) **Multiplication Principle** . Let suppose $\boxed{\text{a process}}$ is to be completed in 3 steps (I followed by II followed by III), and no. of ways to complete step I is 'm', step II → 'n', step III → 'l', then no. of ways to complete the whole process is given by $= 'm \times n \times l'$.

e.g.



No. of ways to come out of this structure = ?

△ → 3

□ → 4

⬠ → 5

Total No. of in which can come out $= 3 \times 4 \times 5$

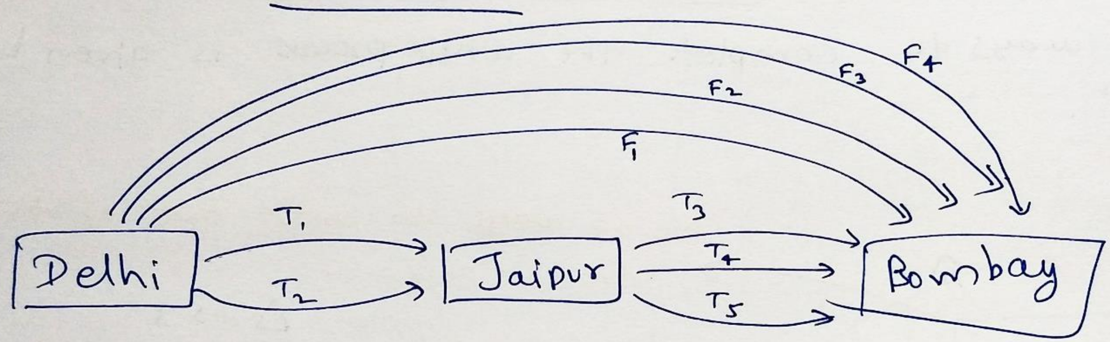
$= 60$

(II) **Addition Principle** :→ If a process is to be completed by 3 options. (each option can complete the whole process)

Let No. of ways to complete I option is m, II → n, III → l,

then no. of ways in which the process can be completed = $m+n+l$.

e.g.



2 options

Flights: 4 ways

Trains: $2 \times 3 = 6$ ways

No. of ways to reach Bombay from Delhi

$= 4 + 6 = 10$ ways

e.g. No. of Different words (4 lettered) (with or without meaning) by the letters of word "ROSE" = ?

(I) Without Repitition of Letters

No. of ways

4	3	2	1
R/O/S/E	O/~~S~~/E	O/~~E~~	O

~~R S O E~~

For Example

ROSE

SORE

ORSE

⋮

R S E O

multiplication rule

$= 4 \times 3 \times 2 \times 1$

$= 24$ ways

(II) with repitition of letters

No. of ways

4	4	4	4
R/O/S/E	R/O/S/E	R/O/S/E	R/O/S/E

~~R O S E~~

Because Repetition is allowed

$= 4 \times 4 \times 4 \times 4$

$= 256$

Exercise 6.1

★
1. Structure
2. Restrictions
3. Repitition

$m \times n$

$m + n$

Q.1

3 Digit Numbers

Available No. = 1, 2, 3, 4, 5

(i) Repitition Allowed.

$5 \times 5 \times 5$ ← Number of ways

1/2/3/4/5

$= 5 \times 5 \times 5$

$= 125$

(ii) Repitition is not allowed

$5 \times 4 \times 3$ ← No. of ways

1/2/3/4/5

$= 5 \times 4 \times 3$

$= 60$

Q.2

3 digit No.

Even No.

1/2/3/4/5/6

Repitition is Allowed

No. of ways → | 6 | 6 | 3 |

1/2/3/4/5/6   2/4/6

$6 \times 6 \times 3$

$= 108$ ways.

(3) → "4 letter Code"

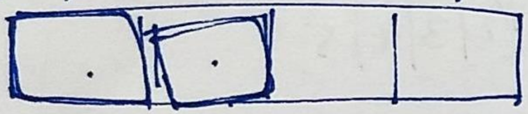
→ By First 10 Letters of English Alphabet.

→ Repitition is not allowed.

(a, b, c ..., j)

No. of ways → $10 \times 9 \times 8 \times 7 = 720 \times 7$

$= \underline{5040 \text{ ways}}$



(a, b, ..., j)

① Structure
② Restrictions
③ Repitition

Q.4

✓ 5 Digit Telephone No.

Using '0' to '9' → 0 1 2 3 4 5 ~~6~~ ~~7~~ 8 9

✓ Each No. starts with 67

Repitition → Not Allowed.

6	7			

$8 \times 7 \times 6$ ← No. of ways

Total No. of ways $= 8 \times 7 \times 6$

$= 56 \times 6$

$= 336$ ✓

Q.5 A coin is tossed 3 times

↓

Outcomes are recorded

How many possible outcomes are there?

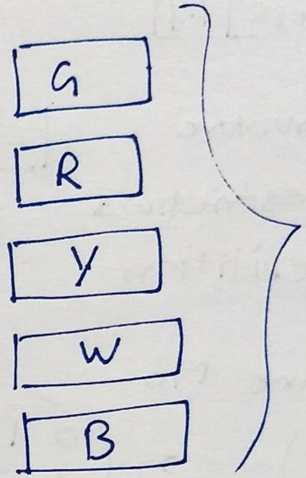
$2 \times 2 \times 2$ ← No. of ways

H/T H/T H/T

$= 2 \times 2 \times 2$

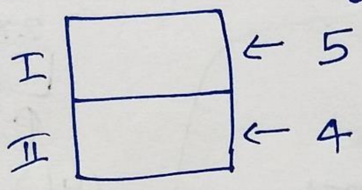
$= 8$

Q.6 5 flags of Different Colours →



No. of ways

Signal →



Total No. of Signals

$= 5 \times 4$

$= 20$ ✓

Note: "Repitition is not Allowed"

Self understood

Factorial Exercise 6.2

Factorial of '5' = $5 \times 4 \times 3 \times 2 \times 1 = 5! = \underline{|5}$

Fatorial of '7' = $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7! = \underline{|7}$

$(-2)!$ ✗

$\left(\frac{1}{2}\right)!$ ✗ → Not Defined

$n! = \underline{|n}$

Definition of factorial of 'n' $(n \in N)$

= the product of first n natural numbers.

$= n.(n-1).(n-2).(n-3).....(2)(1)$

Note

$$\boxed{\text{Factorial of '0'} = 0! = \underline{|0} = 1}$$

→ $0! = 1$

$N \begin{cases} 1! = 1 \\ 2! = 2 \times 1 = 2 \\ 3! = 3 \times 2 \times 1 = 6 \end{cases}$

e.g.

$10! = \lfloor 10$

10 . 9 . 8 . 7 . 6 . 5 . 4 . 3 . 2 . 1

$\leftarrow 9! \rightarrow$

$\leftarrow 8! \rightarrow$

$\leftarrow 6! \rightarrow$

$10! = 10 \times 9 \times \underbrace{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}_{8!}$

$10! = 10 \times 9 \times 8!$

Property of Factorial.

$$n! = n \times (n-1)!$$

$$= n \times (n-1) \times (n-2)!$$

$$= n \times (n-1) \times (n-2) \times (n-3)!$$

$$\vdots$$

$$= n(n-1)(n-2)(n-3) \ldots 3.2.1$$

$n!$

$\frac{8!}{4!}$

$= \frac{8.7.6.5.\cancel{4.3.2.1}}{\cancel{4.3.2.1}}$

$= 8 \times 7 \times 6 \times 5$

$=$ ——

Don't's

$8! + 7! \neq (8+7)! = 15!$ ✗

✗ $\frac{8!}{4!} = 2!$ ✗

Q.1

(i) $8! = 8\times7\times6\times5.4.3.2.1 = 40320$

(ii) $4!-3! = 4\times3.2.1 - 3.2.1$

$= 24-6 = 18$ ✓

Q.2

Is $3!+4! = 7!$? ⟶ wrong

LHS $= 3!+4! = \underbrace{3\times2\times1}+\underbrace{4.3.2.1} = 6+24 = 30$

RHS $= 7! = \underline{7\times6\times5\times4\times3\times2\times1} = 5040$

Q.3

Compute.

$$\frac{8!}{6!\times2!} = \frac{\overset{4}{\cancel{8}}\times7\times\cancel{6!}}{\cancel{6!}\times\cancel{2}\times1} = 28$$ ✓

④ If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x.

$\Rightarrow \frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$

$\Rightarrow \frac{8!}{6!} + \frac{8!}{7!} = x$

$\Rightarrow x = \frac{8 \times 7 \times \cancel{6!}}{\cancel{6!}} + \frac{8 \times \cancel{7!}}{\cancel{7!}} = 56 + 8 = 64$

⑤ Evaluate $\frac{n!}{(n-r)!}$

(i) $n = 6$, $r = 2$

$\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!}$

$= \frac{6!}{4!} = \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 30$

(ii) $n = 9$, $r = 5$

$\frac{n!}{(n-r)!} = \frac{9!}{(9-5)!} = \frac{9!}{4!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}}$

$= 15120$

Theory Before Ex. 6.3

Permutations → Arrangements

Example (16)

$\boxed{\text{Case-I}}$ No. of Arrangements of n-objects (all are different) at r-places (repitition is Allowed).

CAMP

No. of ways → $n \times n \times n \; \cdots \times n$

I II III r^{th}

$= n \times n \times n \times \cdots \times n$ (r - times)

$= n^r$

e.g. CAMP

$4.4.4.4 = 4 \times 4 \times 4 \times 4 \leftarrow$ No. of ways $= 4^4 = 256$

4 places

$4.4.4 = 4 \times 4 \times 4 \leftarrow$ No. of ways $= 4^3 = 64$

3 places

Repitition is Allowed

Case-2 No. of Arrangements of n-objects (all are different) at r-places (repitition is <u>not</u> allowed)

No. of ways = $n \,.\, (n-1) \,.\, (n-2) \,.\, \ldots \,.\, (n-r+1)$

I	II	III	...	r^{th}

$$= n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$$

$$= \frac{n \times (n-1) \times (n-2) \times \ldots \times (n-r+1) \,.\, \boxed{(n-r)\,.\,(n-r-1) \ldots 3.2.1}}{\boxed{(n-r)\,.\,(n-r-1) \ldots 3.2.1}}$$

$$= \boxed{\frac{n!}{(n-r)!} = {}^{n}P_{r}}$$

n → Objects, r → Seats

${}^{n}P_{r}$ → Permutations = No. of ways in which n-different objects can be arranged on r-places without repitition.

<u>Note</u>

$${}^{n}P_{0} = \frac{n!}{(n-0)!} = \frac{\cancel{n!}}{\cancel{n!}} = 1$$

$${}^{n}P_{1} = \frac{n!}{(n-1)!} = \frac{n \times \cancel{(n-1)!}}{\cancel{(n-1)!}} = n$$

$${}^{n}P_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

$0! = 1$

e.g. DAUGHTER → (i) Arrange its all letters at 8 places
(without repitition)

$$= {}^8P_8 = 8! = \boxed{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \text{ ———}$$

DAUGHTER → (ii) Arrange its all letters at 5 places
(without repitition)

8 letters, 5 places

Permutations

$$= {}^{8 \leftarrow \text{Available candidates}}P_{5 \leftarrow \text{Places}}$$

Basics.

8	7	6	5	4

$$= \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{\cancel{3.2.1}}$$

$$= \text{\_\_\_\_\_\_\_\_}$$

Case. (III) No. of Arrangements of n-objects (in which 'p' are alike of 1^{st} kind, 'q' are alike of 2^{nd} kind) at n places $= \boxed{\dfrac{n!}{p!\ q!}}$

kind = type

BANANA → total = 6

B, NN, AAA

NN → 2, AAA → 3

same (Alike)

Reason: BOOK ← BO_1O_2K, BO_2O_1K

$$\frac{4!}{2!}$$

BANANA

↓

$$\frac{6!}{2!\ 3!}$$

Example 16

INDEPENDENCE

total = 12

I
NNN (3)
DD (2)
EEEE (4)
P
C

e.g. NNN $\boxed{IEEEE}$ DDPC

BAG method

Total No. of Arrangements

$$= \frac{12!}{3!\,2!\,4!}$$

(i) words start with 'P'

$\boxed{P}\,\underbrace{\boxed{-----------}}_{11\text{-Places}}$

fixed

$$= \frac{11!}{3!\,2!\,4!}$$

(ii) All the vowels always occur together

Consonants	vowels
NNN, DD, P, C	I, EEEE

IEEEE

Consider as a one object

(I) → Permute N N N D D P C ☐

$$\frac{8!}{3!\,2!}$$

(II) In the Bag → Permute → $\frac{5!}{4!}$

IEEEE

$$\text{Total} = \boxed{\frac{8!}{3!\,2!} \times \frac{5!}{4!}}$$

(iii) All the vowels Never occur together

= total − those cases in which All vowels occur together

$$= \frac{12!}{3!\,2!\,4!} - \frac{8! \times 5!}{3!\,2!\,4!}$$

(iv) words begin with 'I' & end in 'P'

$\boxed{I}\,\underbrace{----------}_{10\text{ Places}}\,\boxed{P}$

Fixed

$$= \frac{10!}{3!\,2!\,4!}$$

1. Structure

2. Restriction

Basic

$\frac{n!}{(n-r)!} = {}^{n}P_{r}$

$n \rightarrow$ Available Candidates (upper suffix)

$r \rightarrow$ seats (lower suffix)

FIRE

$4.3.2.1 = 4!$

$= 24$

$^{4}P_{4} = \frac{4!}{(4-4)!} = \frac{4!}{0!}$

$= \frac{24}{1}$

Q.1 3-digit No. Available 1, 2, 3, 4, 5, 6, 7, 8, 9

By Basics

No. of ways $\rightarrow$ $9 \times 8 \times 7$

$= 9 \times 56$

$= 504$

$^{n}P_{r}$

$= {}^{9}P_{3} = \frac{9!}{6!} = \frac{9.8.7.\cancel{6!}}{\cancel{6!}}$

$= 9 \times 8 \times 7 = 504$

Q.2 4 Digit No. ?

Available 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

No Repitition

× '0'

~~0~~543 $\rightarrow$ 3 digit

Indirect Restriction

No. of ways $\rightarrow$ $9 \times 9 \times 8 \times 7$

1|2|3|4|5|6|7|8|9

0|2|3|4|5|6|7|8|9

Total No. of ways $= 9 \times 9 \times 8 \times 7$

$= 81 \times 56$

$= 4536$

$$\begin{array}{r} 81 \\ \times 56 \\ \hline 486 \\ 405\times \\ \hline 4536 \end{array}$$

Q.3 3 digit No.

Restriction → Even No.

Available
1, 2, 3, 4, ~~5~~, 6, 7

No Repitition

No. of ways → 4 × 5 × 3

2/4/6

Total No. of ways = 4 × 5 × 3

= 60 ✓

Basics.

^nP_r

$^5P_2 \times 3$

2/4/6

$= {}^5P_2$

Total No. of ways $= {}^5P_2 \times 3$

$= \frac{5!}{3!} \times 3$

$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}} \times 3$

$= 60$

Q. 4 4 digit No.

Available
1, 2, 3, 4, 5

No Repitition

(i) Total No. of 4 Digit No.

→ 5 . 4 . 3 . 2

1/2/3/4/5

No Restriction

= 5×4×3×2

= 120 ✓

(ii) Total No. of Even No. (4 Digit.)

RESTRICTION

→ 2 × 3 × 4 × 2

2/4

= 2×3×4×2

= 48

$\boxed{Q.5}$ Available → 8 persons

multiply

and → 'x'

Chairman ? 8

Vice Chairman ? 7

Total No. of ways $= 8 \times 7$

$= 56$

$\boxed{Q.6}$ Find n, if $\boxed{{}^{n-1}P_3 : {}^{n}P_4 = 1 : 9}$

$\checkmark \boxed{{}^{n}P_r = \dfrac{n!}{(n-r)!}}$ $\checkmark \boxed{n! = n.(n-1)! = n.(n-1).(n-2)! = \ldots}$

Given $\dfrac{{}^{n-1}P_3}{{}^{n}P_4} = \dfrac{1}{9} \Rightarrow \dfrac{\dfrac{(n-1)!}{(n-1-3)!}}{\dfrac{n!}{(n-4)!}} = \dfrac{1}{9}$

$\Rightarrow \dfrac{\dfrac{(n-1)!}{\cancel{(n-4)!}}}{\dfrac{n!}{\cancel{(n-4)!}}} = \dfrac{1}{9}$

$\Rightarrow \dfrac{(n-1)!}{n!} = \dfrac{1}{9}$

$\Rightarrow \dfrac{\cancel{(n-1)!}}{n.\cancel{(n-1)!}} = \dfrac{1}{9}$

$\Rightarrow \dfrac{1}{n} = \dfrac{1}{9}$

$\Rightarrow \boxed{n = 9}$ ✓

Q.7 Find 'r'.

^nP_r

$0 \le r \le n$

(i) $^5P_r = 2 \cdot {}^6P_{r-1}$

$\Rightarrow \dfrac{5!}{(5-r)!} = 2 \cdot \dfrac{6!}{[6-(r-1)]!}$

$\Rightarrow \dfrac{5!}{(5-r)!} = 2 \times \dfrac{6!}{(7-r)!}$

$\Rightarrow \dfrac{\cancel{5!}}{\cancel{(5-r)!}} = 2 \times \dfrac{6 \times \cancel{5!}}{(7-r)(6-r)\cancel{(5-r)!}}$

$\Rightarrow (7-r) \cdot (6-r) = 12$

$\Rightarrow 42 - 13r + r^2 = 12$

$\Rightarrow r^2 - 13r + 30 = 0$

$\Rightarrow r^2 - 10r - 3r + 30 = 0$

$\Rightarrow r(r-10) - 3(r-10) = 0$

$\Rightarrow (r-3)(r-10) = 0$

$\boxed{r = 3, 10}$

$r = 3$ ✓ $\quad {}^5P_3 = 2 \times {}^6P_{3-1}$ ✓

$r = 10$ ✗ $\quad {}^5P_{10} = 2 \times {}^6P_9$ ✗

$10 > 5$ ✗

(ii) $^5P_r = {}^6P_{r-1}$

$\Rightarrow \dfrac{5!}{(5-r)!} = \dfrac{6!}{[6-(r-1)]!}$

$\Rightarrow \dfrac{\cancel{5!}}{\cancel{(5-r)!}} = \dfrac{6 \times \cancel{5!}}{(7-r) \cdot (6-r) \cdot \cancel{(5-r)!}}$

$\Rightarrow (7-r)(6-r) = 6$

$\Rightarrow 42 - 13r + r^2 = 6$

$\Rightarrow r^2 - 13r + 36 = 0$

$\Rightarrow r^2 - 9r - 4r + 36 = 0$

$\Rightarrow r(r-9) - 4(r-9) = 0$

$\Rightarrow (r-4)(r-9) = 0$

$r = 4, 9$

$r = 4$ ✓ $\quad {}^5P_4 = {}^6P_{4-1}$ ✓

$r = 9$ ✗ $\quad {}^5P_9 = {}^6P_{9-1}$ ✗

Q.8 EQUATION

$$^{n}P_r = \frac{n!}{(n-r)!}$$

Available → All the letters.

Repitition → No.

E Q U A T I O N

8 . 7 . 6 . 5 . 4 . 3 . 2 . 1

By Basics $= 8 \times \underline{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 8 \times 5040$

$= 40320$

By $^{n}P_r$ — n ← Available candidates, r ← No. of seats. $= \frac{n!}{(n-r)!}$

$= {}^{8}P_8 = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{40320}{1}$

Q.9 MONDAY

Repitition ↓ Not Allowed

(i) 4 letters at a time.

Basics.

6 . 5 . 4 . 3

M | O | N | D | A | Y

Total No. of words $= 6.5.4.3$

$= 360.$

$^{n}P_r$ — n → Available, r → Seats

$= {}^{6}P_4$

$= \frac{6!}{2!}$

$= \frac{720}{2}$

$= 360$

MONDAY

(ii) all letters are used at a time.

6 seats.

Basic

6 . 5 . 4 . 3 . 2 . 1

M | O | N | D | A | y

Total No. of words $= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

$\boxed{{}^{n}P_{r}}$ n → Available, r → seats.

$= {}^{6}P_{6}$

$= \frac{6!}{(6-6)!}$

$= \frac{6!}{0!} = \frac{720}{1}$

MONDAY → Vowels (O, A) ; (M, N, D, y)

(iii) all letters are used but first letter is a vowel.

Restriction ← Priority.

Basics.

2 . 5 . 4 . 3 . 2 . 1

O/A

A | M | N | D | y

No. of words

$= 2.5.4.3.2.1$

$= 2 \times 120$

$= 240$

${}^{n}P_{r}$

$2 \times {}^{5}P_{5}$ — 5 candidates, 5 seats

O/A

No. of words

$= 2 \times {}^{5}P_{5}$

$= 2 \times \frac{5!}{0!}$

$= 2 \times \frac{120}{1} = 240$

n objects

p-objects → same$_1$

q-obj. → same$_2$

No. of permutations $= \dfrac{n!}{p!\,q!}$

* Q.10

MISSISSIPPI → M SSSS PP, IIII

No. of words in which all I's donot occur together ? = Total No. of words ? − No. of words in which all I's occur together ?

$= \dfrac{11!}{4!\,2!\,4!} - \dfrac{8!}{4!\,2!} = $ Answer.

M | SSSS | PP | IIII → 11

$= \dfrac{11!}{4!\,2!\,4!}$ ← Total.

$4!$ ← SSSS, $2!$ ← PP, $4!$ ← IIII

Bag Method

M | SSSS | PP | $\boxed{IIII}$ → 1-object

$= \dfrac{8!}{4!\,2!}$ ✓

$4!$ ← SSSS, $2!$ ← PP

* → Example 16

Q.11 PERMUTATIONS $\rightarrow$ total = 12

P | E | R | M | U | A | I | O | N | S | TT

(i) Start with 'P' & End with 'S'

P											S

Fixed $\underbrace{\qquad\qquad\qquad}_{\text{10 seats}}$ Fixed

Available $\rightarrow$ E, R, M, U, A, I, O, N, TT

No. of permutations = $\dfrac{10!}{2!}$ ← TT

$= \dfrac{10!}{2!}$ = Answer.

(ii) Vowels are together → BAG METHOD

Vowels $\rightarrow$ E, U, A, I, O

Consonants $\rightarrow$ P, R, M, N, S, TT

$$\underbrace{\underset{1}{P},\ \underset{2}{R},\ \underset{3}{M},\ \underset{4}{N},\ \underset{5}{S},\ \underset{6}{T},\ \underset{7}{T},\ \underset{8}{(AEIUO)}}$$

I Step. Arrange these only, $= \dfrac{8!}{2!}$ ← TT

II step we must also consider the Permutations of $\boxed{AEIOU}$ inside the Bag. $= 5!$

Total No. of words in which all vowels occur together

$= \dfrac{8!}{2!} \times 5! =$ ______ = Answer.

(iii) "PERMUTATIONS"

↳ there are always 4 letters between P & S?

for e.g.

- - P _ _ _ _ S - - - -

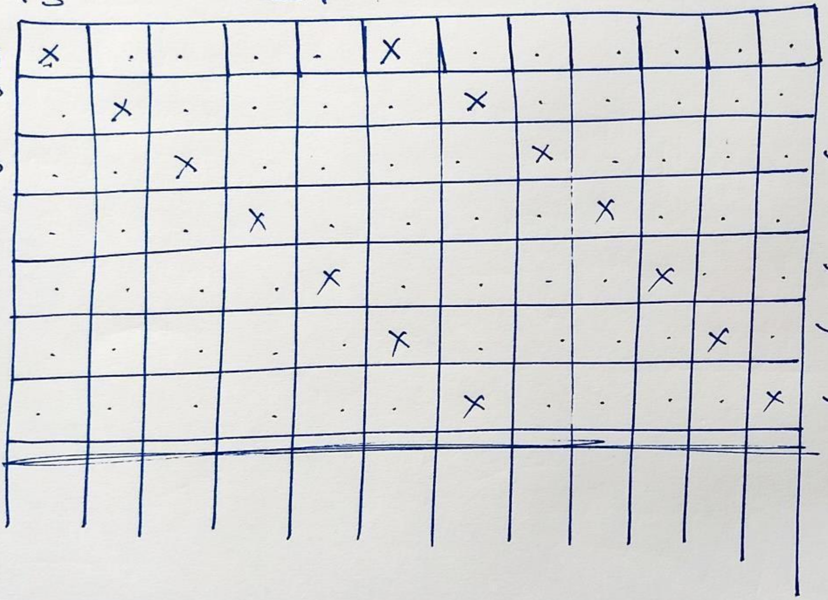
S - - - - P

~~P~~, ~~S~~, $\underbrace{\text{E, R, M, U, A, I, O, N, T, T}}_{10}$

T, T → Same.

$2\begin{cases} P \text{ ——— } S \\ S \text{ ——— } P \end{cases}$

Similarly



{ P....S ; S....P }

Remaining seats 10

$2 \times \frac{10!}{2!}$ → TT

Total No. of ways

$$= 7 \times 2 \times \left(\frac{10!}{2!}\right) = \text{Answer}$$

. . X . . . . X . .

This pattern can shift 7 times.

P . . . . S / S . . . . P

Remaining

Ex 6.4

COMBINATIONS | ${}^{n}C_{r}$ | EXAMPLE-19

↓

Selection

Permutations : No. of permutations (arrangement) of n-objects at 'r' places

$$= {}^{n}P_{r} = \frac{n!}{(n-r)!} \quad (0 \le r \le n)$$

Combinations : No. of combinations (selection) of r-objects out of n-objects.

$$= {}^{n}C_{r} = ? \quad (0 \le r \le n) = \frac{n!}{(n-r)!\, r!}$$

$${}^{5}P_{3} = {}^{5}C_{3} \times 3!$$

$$\Rightarrow \frac{5!}{(5-3)!\, 3!} = {}^{5}C_{3}$$

e.g. 5-Persons → 3 seats (Arrange) → permutations

(i) 5. 4. 3 No. of Permutations $= 5.4.3 = {}^{5}P_{3}$

(ii) Select '3' out of '5' & Arrange '3' persons on 3 seats. = No. of ways

$\hookrightarrow {}^{n}C_{r} = {}^{5}C_{3}$ 3.2.1 $= 6 = 3!$ $= {}^{5}C_{3} \times 3!$

Properties of ^nC_r

$n \to$ upper suffix.
$C \to$ Combination (selection)
$r \to$ lower Suffix.

① $$\boxed{{}^nC_r = \frac{n!}{(n-r)!\, r!}}$$

$$\boxed{{}^nC_r = \frac{{}^nP_r}{r!}} \Rightarrow \boxed{{}^nP_r = {}^nC_r \cdot r!}$$

② $$\boxed{{}^nC_0 = 1} \qquad {}^nC_0 = \frac{n!}{(n-0)!\, 0!} = \frac{\cancel{n!}}{\cancel{n!} \times 1} = 1$$

$$\boxed{{}^nC_n = 1} \qquad {}^nC_n = \frac{n!}{(n-n)!\, n!} = \frac{\cancel{n!}}{0! \times \cancel{n!}} = 1$$

$$\boxed{{}^nC_1 = n} \checkmark$$

$$\boxed{{}^nC_{n-1} = n} \checkmark$$

Properties of ^nC_r

(3) $\boxed{^nC_r = {}^nC_{n-r}}$

$LHS = {}^nC_r = \dfrac{n!}{(n-r)!\, r!}$ ✓

$RHS = {}^nC_{n-r} = \dfrac{n!}{[n-(n-r)]!\,(n-r)!}$

$= \dfrac{n!}{r!\,(n-r)!}$ ✓

e.g. $^{10}C_3 = {}^{10}C_{10-3}$

$\boxed{^{10}C_3 = {}^{10}C_7}$ $\quad 3+7=10$

e.g. $\boxed{^{12}C_4 = {}^{12}C_8}$ $\quad 4+8=12$

(4) If $^nC_a = {}^nC_b$ then

(i) $a=b$ or

(ii) $a+b=n$

e.g. $^nC_7 = {}^nC_8$

$n = ?$

$7+8=n$

$15=n$

Properties of ${}^{n}C_r$

(5) $\boxed{{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r}$

$\boxed{{}^{7}C_3 + {}^{7}C_4 = {}^{8}C_4}$

$\boxed{{}^{n}C_r = \frac{n!}{(n-r)!\,r!}}$

Proof: LHS $= {}^{n}C_r + {}^{n}C_{r-1}$

$$\text{LHS} = \frac{n!}{(n-r)!\cdot r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\cdot r\cdot(r-1)!} + \frac{n!}{(n-r+1)\cdot(n-r)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\cdot\left\{\frac{1}{r} + \frac{1}{n-r+1}\right\}$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\cdot\left\{\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right\}$$

$$= \frac{(n+1)!}{(n-r+1)!\;r!}$$

RHS $= {}^{n+1}C_r$

$$= \frac{(n+1)!}{(n+1-r)!\;r!}$$

LHS = RHS.

Fundamental Principle of Counting :

$\boxed{\text{Before Example (19)}}$

① **Multiplication Rule** (&) (and)

Project.

$\boxed{\text{I Step}}$ & $\boxed{\text{II Step}}$ & $\boxed{\text{III Step}}$

$$= m \times n \times l$$

② **Addition Rule** (or) (या)

Project.

$\boxed{\text{I option}}$ या $\boxed{\text{II-option}}$ या $\boxed{\text{III-option}}$

$$m + n + l$$

For example (19)

52

Red 26 — Black 26

HEART ♡	DIAMOND ◊	SPADE ♤	CLUB ♧
13	13	13	13
A	A	A	A
K	K	K	K
Q	Q	Q	Q
J	J	J	J
10	10	10	10
9	9	9	9
8	8	8	8
7	7	7	7
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2

Face Cards (K, Q, J)

Honor Cards (A, K, Q, J)

Example - 19

No. of ways of choosing 4 cards from 52 playing cards $= {}^{52}C_4$

(i) 4 cards are of the same suit = either Heart or Dia. or Spade or Club

$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

(ii) 4 cards belong to the 4 different suits = 1 from Heart & 1 from Diamond & 1 from Spade & 1 from club

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$= (13)^4$

(iii) are face cards $\rightarrow$ 12 = total.

$= {}^{12}C_4 = \frac{12!}{8!\ 4!}$

(iv) 2 are red $\rightarrow$ 26 & 2 are black $\rightarrow$ 26

$= {}^{26}C_2 \times {}^{26}C_2$

$= \left({}^{26}C_2\right)^2$

(v) Cards are of the same colour

either Red Cards $\rightarrow$ 26 or Black $\rightarrow$ 26

$= {}^{26}C_4 + {}^{26}C_4$

$= 2 \times {}^{26}C_4$

Revision:

① $^nC_r = \frac{n!}{(n-r)!\, r!}$

② $^nC_r = {^nC_{n-r}} \longrightarrow {^{13}C_2} = {^{13}C_{11}}$

③ $^nC_a = {^nC_b}$ → (i) $a = b$ or → (ii) $\underline{a+b=n}$ ☆

Extra prop.

$$\underline{^nC_r + {^nC_{r-1}} = {^{n+1}C_r}}$$

$\boxed{Q.1}$ $^nC_8 = {^nC_2}$; Find $^nC_2$

By property ③.

$8 \neq 2$

then $8+2 = n$

$\Rightarrow$ $n = 10$

$^nC_2 = {^{10}C_2}$

$= \frac{10!}{8!\ 2!}$

$= \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!} \times \cancel{2} \times 1} = 45$ ✓

Q.2 (i) ${}^{2n}C_3 : {}^{n}C_3 = 12 : 1$

$${}^{n}C_r = \frac{n!}{(n-r)!\,r!}$$

$$\Rightarrow \frac{\left[\dfrac{(2n)!}{(2n-3)!\,\cancel{3!}}\right]}{\left[\dfrac{n!}{(n-3)!\,\cancel{3!}}\right]} = \frac{12}{1}$$

$$\Rightarrow \frac{\dfrac{(2\cancel{n})(2n-1)\overset{2}{\cancel{(2n-2)}}\cdot\cancel{(2n-3)!}}{\cancel{(2n-3)!}}}{\dfrac{\cancel{n}\cdot\cancel{(n-1)}(n-2)\cdot\cancel{(n-3)!}}{\cancel{(n-3)!}}} = \frac{12}{1}$$

$$\Rightarrow \frac{\cancel{4}(2n-1)}{n-2} = \frac{\cancel{12}^{\,3}}{1.}$$

$$\Rightarrow 2n-1 = 3n-6$$

$$\Rightarrow 6-1 = 3n-2n \Rightarrow \boxed{5 = n}$$

(ii) ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1.$

$$\Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1.}$$

$$\Rightarrow 8n-4 = 11n-22$$

$$\Rightarrow 22-4 = 11n-8n$$

$$\Rightarrow \cancel{3}n = \cancel{18}\,6$$

$$\boxed{n=6}$$

<u>Revision</u>, ① ^nC_r = No. of combinations (selection) of r-objects taken from <u>n-objects</u>.

$(0 \le r \le n)$

② $^nC_r = \frac{n!}{(n-r)!\, r!}$

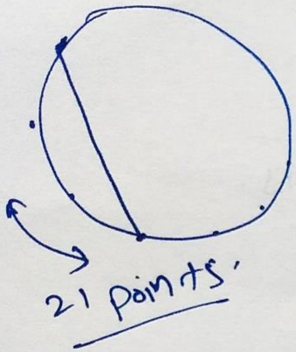
③ <u>multiplication Rule</u> → (&) $\underbrace{\text{I Step}}_{m} \;\&\; \underbrace{\text{II Step}}_{n}$

$m \times n = m \cdot n$

Addition Rule (or) $\underbrace{\text{I option}}_{m}$ or $\underbrace{\text{II - option}}_{n}$

$m + n = m+n$

Q.3 21 points on a ~~Circle~~ Circle



→ one chord is made by joining two particular points of circle.

→ So we have to select 2 points to make a chord. (out of 21 points.)

No. of chords = No. of ways to select 2 points of the circle (out of 21 points.)

$$= {}^{21}C_2$$

$$= \frac{21!}{19!\,2!} = \frac{21 \times \cancel{20}^{10} \times \cancel{19!}}{\cancel{19!} \times \cancel{2} \times 1} = 210$$

$\boxed{Q.4}$ from $\underbrace{5 \text{ Boys}}$ & $\underbrace{4 \text{ Girls}}$

Select a team → $\boxed{3 \text{ Boys}}$ & $\boxed{3 \text{ Girls}}$

No. of ways to select a team $= {}^5C_3 \times {}^4C_3$

$= \frac{5!}{2!\,3!} \times \frac{4!}{1!\,3!}$

$= \frac{5.\cancel{4}^{2}.\cancel{3.2.1}}{\cancel{2}.1.\cancel{3.2.1}} \times \frac{4.\cancel{3.2.1}}{1 \times \cancel{3.2.1}}$

$= 5 \times 2 \times 4$

$= 40$

$\boxed{Q.5}$ from $\underbrace{6 \text{ Red}}$, $\underbrace{5 \text{ white}}$, $\underbrace{5 \text{ Blue}}$

Select total 9-Balls → $\boxed{3R}$ & $\boxed{3W}$ & $\boxed{3B}$

No. of ways to select 9 balls (if 3 balls of each color) $= {}^6C_3 \times {}^5C_3 \times {}^5C_3$

$= \frac{6!}{3!\,3!} \times \frac{5!}{2!\,3!} \times \frac{5!}{2!\,3!}$

$= \frac{6.5.4.\cancel{3!}}{\cancel{3!}\,3!} \times 10 \times 10$

$= 20 \times 10 \times 10$

$= 2000$

Revision $^{n}C_{r}$ = no. of ways of selecting r-objects out of n-objects.

$$^{n}C_{r} = \frac{n!}{(n-r)!\, r!}$$

$\boxed{Q.6}$ Total 52 Cards $\xrightarrow{\text{Select}}$ 5 Cards.

4 A & 48 other Cards.

1 Ace & 4 other cards

Exactly

Select.

Select.

Total No. of ways $= \left(^{4}C_{1}\right) \times \left(^{48}C_{4}\right)$

$= {}^{4}C_{1} \times {}^{48}C_{4}$

$= 4 \times {}^{48}C_{4}$

$\boxed{Q.7}$

from total 17 players → $\boxed{\text{5 Bowlers}}$ & $\boxed{\text{12 others}}$ ← Available

Select Cricket team of 11 players → $\boxed{\text{4 Bowlers}}$ Exactly & $\boxed{\text{7 others}}$ ← Desire.

Total No. of ways of selecting a cricket team of 11-players $= {}^5C_4 \times {}^{12}C_7 = {}^5C_4 \cdot {}^{12}C_7$

$$= \frac{\cancel{5!}}{1!\,4!} \cdot \frac{12!}{\cancel{5!}\,7!}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9}^{3} \cdot \cancel{8} \cdot \cancel{7!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{7!}}$$

$$= 12 \times 11 \times 10 \times 3$$

$$= 3960$$

(8)

Bag: 5 Black Balls, 6 Red Balls

No. of ways of selecting $\boxed{2 \text{ Black}}$ & $\boxed{3 \text{ Red}}$ Balls

$$= {}^5C_2 \times {}^6C_3$$

$$= \frac{5!}{3!\,2!} \times \frac{6!}{3!\,3!}$$

$$= \frac{5.\cancel{4}^{2}.\cancel{3}.\cancel{2}.\cancel{1}}{\cancel{3}.\cancel{2}.\cancel{1}.\cancel{2}.1} \times \frac{\cancel{6}.5.4.\cancel{3!}}{\cancel{3!}\,\cancel{3.2.1}}$$

$$= 10 \times 20 = 200$$

(9) Compulsory 2 specific → Fixed by Question itself.

Available 9 Courses → $\boxed{\text{2 Specific Compulsory}}$ + $\boxed{\text{7 others}}$

Choose 5 Courses → $\boxed{\text{2 Specific Compulsory}}$ & $\boxed{\text{3 others optional}}$

${}^2C_2 = 1$

No. of ways of choosing the programme $= 1 \times {}^7C_3$

$$= 1 \times {}^7C_3$$

$$= 1 \times \frac{7!}{4!\,3!}$$

$$= \frac{7.6.5.\cancel{4!}}{\cancel{4!}\,3!}$$

$$= 35$$

Q.1 'DAUGHTER' Available

Vowels — ③ A, U, E

Consonants — ⑤ D, G, H, T, R

^nC_r = Selection $= \frac{n!}{(n-r)!\,r!}$

Desired: 2 vowels & 3 consonants.

$^3C_2$ $^5C_3$

Combination (Selection)

Repetition → No.

Different words → $5.4.3.2.1 = 5! = 120$ (Arrangement)

Permutation (Arrangement)

Total No. of words = No. of ways of Selection (Combination) × No. of ways of Arrangement (Permutation)

$= {}^3C_2 \times {}^5C_3 \times 120$

$= \frac{3!}{1!\,2!} \times \frac{5!}{2!\,3!} \times 120 = \frac{3\times\cancel{2\times1}}{1\times\cancel{2\times1}} \times \frac{5.\cancel{4}^{2}.\cancel{3.2.1}}{\cancel{2}.1.\cancel{3.2.1}} \times 120 = 3600$

Q.2 'EQUATION'

All the letters

Repetition
↓
Not Allowed

Vowels Together
'EUAIO'

Consonants together
'QTN'

Bag_V

AEIOU

Bag_C

QTN

For e.g.

AEIOU QTN →

EIAUO TNQ →

TNQ EIAUO

Arrangement of Bags = 2

V C

C V

Arrangement of vowels in their own Bag = 5!

AEIOU

5.4.3.2.1

Arrangement of Consonants in their own Bag = 3!

QTN

3.2.1

Total no. of words = $2 \times 5! \times 3!$

$= 2 \times 120 \times 6 = 1440$

$$ {}^{n}C_r = \frac{n!}{(n-r)!\,r!} $$

Q.3 $\boxed{\text{Total Available} = 9 \text{ Boys } \& \ 4 \text{ Girls} \quad \text{Form Committee by 7 members}}$

(i) Exactly 3 Girls

Committee 7-member

& 3 G → selection ${}^{4}C_3$

4 B → ${}^{9}C_4$

$= {}^{4}C_3 \times {}^{9}C_4$

Girls Boys

$= \frac{4!}{1!\,3!} \times \frac{9!}{5!\,4!}$

$= \frac{4 \cdot \cancel{3!}}{1 \times \cancel{3!}} \times \frac{9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6}^{2} \cdot \cancel{5!}}{\cancel{5!}\ \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$

$= 4 \times \underbrace{9 \cdot 7 \cdot 2}_{126}$

$= 504$

(ii) atleast 3 Girls

(कम से कम) → 3, 4 G.

3 G / 4 B or 4 G / 3 B

$= \underline{{}^{4}C_3 \times {}^{9}C_4} + \underline{{}^{4}C_4 \times {}^{9}C_3}$

$= \frac{4!}{1!\,3!} \times \frac{9!}{3!\,4!} + \frac{4!}{0!\,4!} \times \frac{9!}{6!\,3!}$

$= \boxed{\text{Answer.}}$

(iii) atmost 3 girls

(ज्यादा से ज्यादा)

3 G / 4 B or 2 G / 5 B or 1 G / 6 B or 0 G / 7 B

$\underbrace{{}^{4}C_3 \times {}^{9}C_4} + \underbrace{{}^{4}C_2 \cdot {}^{9}C_5} + \underbrace{{}^{4}C_1 \cdot {}^{9}C_6} + \underbrace{{}^{4}C_0 \cdot {}^{9}C_7}$

$= \boxed{\text{Answer}}$

$\boxed{Q.4}$ 'EXAMINATION'

In alphabetical order → $\underbrace{A\,A\,E\,I\,I\,M\,N\,N\,O\,T\,X}$ → total letters = 11

Before the first word starting with 'E'

Dictionary.

A - - - - - - - - } → ??

E - - - - - - - -

I ________

M ________

⋮

X ________

Alike

total = n

p → same₁,

q → same₂

→ Permutation $= \dfrac{n!}{p!\,q!}$

No. of words starting ~~the~~ with the letter 'A'

↑ Restriction

Priority

$= \boxed{A}\,|\,.\,|\,.\,|\,.\,|\,.\,|\,.\,|\,.\,|\,.\,|\,.\,|\,.\,|\,.$

Fixed $\qquad$ 10 Seats.

A, E, I I, M N N, O, T, X

alike $\qquad$ alike

$= \dfrac{10!}{2!\,2!}$

$= \dfrac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\times 2}$

$= 10\times 10\times 72\times \overbrace{42\times 3}^{126} = 907200$

$$\begin{array}{r} 126 \\ \times 72 \\ \hline 252 \\ 882\times \\ \hline 9072 \end{array}$$

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(5). Available = 0, 1, 3, 5, 7, 9

6 - Digit No.

Fixed.

Priority

"Divisible" by 10 ← Restriction (Priority)

(Last Digit = 0)

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 5!$

$\boxed{\text{No. of ways} = 120}$

(6) Available: $\boxed{5 \text{ Vowels}}$ $\boxed{21 \text{ Consonants}}$

Select

Desired: $\boxed{2 \text{ Vowels}}$ & $\boxed{2 \text{ Consonants}}$

only selection

$^5C_2 \times {}^{21}C_2$

Arrange: AERT, RATE

(2V, 2C)

After Selection Available → 4 letters

$4 \cdot 3 \cdot 2 \cdot 1$

Total No. of words $= \underbrace{4 \times 3 \times 2 \times 1}_{\text{Arrange}} \times \underbrace{{}^5C_2 \cdot {}^{21}C_2}_{\text{selection}}$

$= 4 \cdot 3 \cdot 2 \cdot 1 \cdot \dfrac{5!}{3!\,2!} \times \dfrac{21!}{19!\,2!}$

Q.7

12 Questions

Part I 5Q.

1, 2, 3, 4, 5

Part II 7Q.

6, 7, 8, 9, 10, 11, 12

selection

^nC_r

Attempt 8 Questions

at least 3 from each part

Case-I. 3Q (I) & 5Q (II) $= \boxed{^5C_3 \times {^7C_5}}$

or

Case-II. 4Q (I) & 4Q (II) $+ \boxed{^5C_4 \times {^7C_4}}$

or

Case-III. 5Q (I) & 3Q (II) $+ \boxed{^5C_5 \times {^7C_3}}$

= Answer

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

=

$\boxed{Q.8}$ 52 Cards Total $\xrightarrow{\text{Select}}$ 5 Cards

$\boxed{\text{4 Kings \& 48 others}}$ $\boxed{\text{1 King \& 4 other Cards}}$

Select.

Exactly

No. of ways $= {}^{4}C_{1} \times {}^{48}C_{4}$

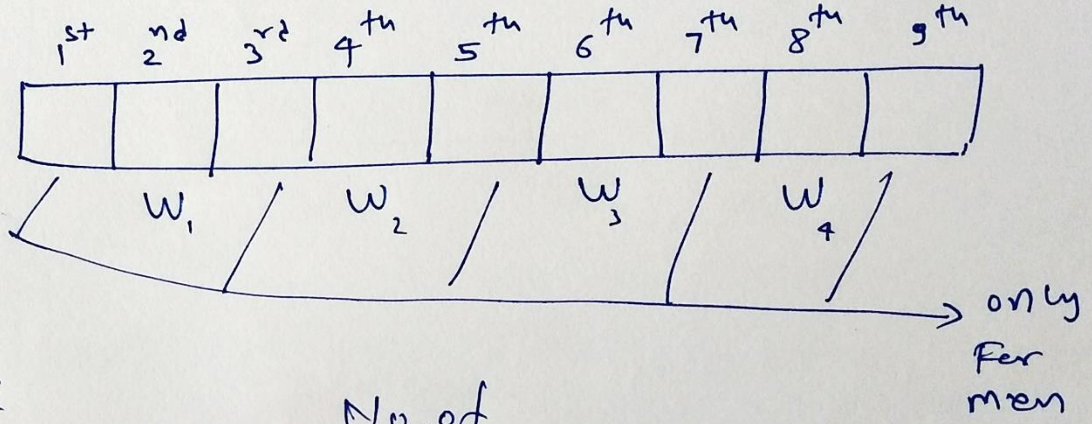
$= $ Answer.

$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Q.9 Available → 5 men , 4 women

$\boxed{\text{Women occupy Even places}}$

Total seats = 9



No. of total ways = No. of Arrangements of women × No. of Arrangements of men

2nd 4th 6th 8th | 1st 3rd 5th 7th 9th

$$= (4 \times 3 \times 2 \times 1) \times (5 \times 4 \times 3 \times 2 \times 1)$$

$$= 4! \times 5!$$

$$= 24 \times 120$$

$$= 2880$$

Q.10

25 Students

Party → 10 Students

Special '3' | 22 others

Case (I)

$\underbrace{\ \ \ \ \ \ \ }_{\text{Either all of them}}$ ------- (7) others

$= {}^{3}C_{3} \times {}^{22}C_{7}$

special → 1

$= 1 \times {}^{22}C_{7}$

+

Case (II)

$\overbrace{\text{----- -----}}^{\text{all 10 other}}$

or none of them

$= {}^{22}C_{10}$

+

$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Total No. of ways $= {}^{22}C_{7} + {}^{22}C_{10}$

$$= \frac{22!}{15!\,7!} + \frac{22!}{12!\,10!}$$

— Answer.

Q.11 ASSASSINATION $\longrightarrow$ AAA, II, NN, O, T, SSSS,

All the S's are together $\rightarrow$ Bag method.

A A A, I, I, N, N, O, T, SSSS

10 objects $\rightarrow$ Permutations

No. of words $= \dfrac{10!}{3!\ 2!\ 2!}$

$3! \rightarrow$ AAA, $2! \rightarrow$ 2I, $2! \rightarrow$ NN

$$= \frac{10 \times 9 \times 8 \times 7 \times \cancel{6} \times 5 \times \cancel{4} \times 3 \times 2 \times 1}{\cancel{3 \times 2} \times 1 \times \cancel{2} \times 1 \times \cancel{2} \times 1}$$

$= 10 \times 10 \times 72 \times 21$

$= 151200$

n – total
p → alike$_1$
q → alike$_2$
$\rightarrow \dfrac{n!}{p!\,q!}$

$$\begin{array}{r} 72 \\ \times\ 21 \\ \hline 72 \\ 144\times \\ \hline 1512 \end{array}$$